The Binomial Distribution

Error Words on the BSC

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Words on the BSC

- Send an n-bit word $\mathbf{x}$ over BSC(p)
- The received word is $\mathbf{R} = \mathbf{x} \oplus \mathbf{E}$
  - where $\mathbf{E}$ is a random error vector

Problem

Let $T = w(\mathbf{E})$ be the number of bit errors.
Describe the probability distribution of T.

We will solve the problem in two steps.

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Distribution of the error vector

- The error word **E** is a stochastic variable.
- We can start with the probability distribution of **E**.

Exercise

What is the probability that $\mathbf{E} = (0100110)$*?*

Solution

Distribution of the error vector

$$P(\mathbf{E} = (0100110)) =$$

Probability of an error word

The probability of a given error word $\mathbf{e}$ *depends only on the number of bit errors* $w(\mathbf{e})$.

$$P(\mathbf{E} = \mathbf{e}) = p^t(1-p)^{n-t}, \tag{1}$$

$$\text{where } t = w(\mathbf{e}). \tag{2}$$

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Counting possible error words

The probability of a given error word depends only on the number of bit errors.

Exercise

How many n-bit words exist with Hamming weight t?

This is a fundamental counting problem.

Solution

Counting possible error words

- How many n-bit (error) words exist with Hamming weight t?
- Choose t error positions out of n possible.
- How?

This is what the binomial coefficient is for...

$$\binom{n}{t} = \frac{n!}{t!(n-t)!}. \tag{3}$$

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Solution

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The probability of t errors

What is the probability $P(T = t)$?

- Multiply
  1. the probability of a given t-error word
  2. the number of possible t-error words

$$P(T = t) = \binom{n}{t} p^t (1-p)^{n-t}$$

Closure

- Let X be the number of successes in n Bernoulli trials with success probability p
- X is binomially distributed with probability p
- We write $X \sim B(n, p)$

- A bit transmission on BSC is a Bernoulli trial
- The number X of bit errors on an n-bit word
  - is binomially distributed
- We write $X \sim B(n, p)$

Exercise

What other examples of binomially distributed variables can you find? Review binomial distributions in the textbook (Frisvold and Moe).