Point Estimation

The inaccuracy of estimates

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The Monte Carlo Experiment

Recall the Monte Carlo simulator.

- Test the system n times
  - Record the number of errors X
  - Report the error rate $R = X/n$
- Estimate the error probability p_e

Point Estimator

Definition

- A (point) estimator of a parameter θ
  1. is a function of the observed data
  2. which can be used to estimate θ
- Write $\hat{\theta}$ for the estimator
- It follows that
  1. $\hat{\theta}$ is a stochastic variable
  2. $\hat{\theta} \approx \theta$ with high probability

Error Rate

- Error count: $X \sim B(n, p_e)$
- Error rate: X/n
- Estimator: $\hat{p}_e = X/n$
- $E(\hat{p}_e) = p_e$

Definition (Unbiased estimator)

If $E(\hat{\theta}) = \theta$, we say that $\hat{\theta}$ is an unbiased estimator of θ.

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Probability distribution

- $\hat{p}_e$ is random
  - it has a variance and standard deviation $\sigma_{\hat{p}_e}$
- Estimation error $|\hat{p}_e - p_e|$
  - $\sim$ 32% of time: $|\hat{p}_e - p_e| > \sigma_{\hat{p}_e}$
  - $\sim$ 4.5% of time: $|\hat{p}_e - p_e| > 2\sigma_{\hat{p}_e}$
  - $\sim$ 0.25% of time: $|\hat{p}_e - p_e| > 3\sigma_{\hat{p}_e}$
- (This is assumes large numbers or normal distribution.)

A good estimator needs a low variance.

Exercise

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Suppose you are testing a system with error probability of 0.01*. How many trials do you need to make your estimator* $\hat{p}_e$ *fall between* 0.011 *and* 0.009 99.75% *of the time?*