The Expected Value

The Binomial Distribution

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The Binomial Distribution

$$P(T = t) = \binom{n}{t} p^t (1 - p)^{n-t}$$

Problem

What is the expected value $E(T)$ where the probability distribution of T is given above.

Toy case $N = 1$

- Consider a single Bernoulli trial.
  - $X \sim B(1, p)$
- What is the expected value $E(X)$
  - $E(X) = \sum_x x \cdot P(X = x)$

Outcome	X	Probability p'	$p' \cdot X$
Success	1	p	p
Failure	0	$1 - p$	0
		Sum	p

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General case $N = ?$

- Binomial distribution $Y \sim B(n, p)$
- $Y = X_1 + X_2 + \ldots + X_n$
  - Each $X_i \sim B(1, p)$
  - Independent X_i
- $E(Y) = \sum_{i=1}^{n} E(X_i) = n \cdot E(X) = n \cdot p$

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Summary

- Binomial distribution $X \sim B(n, p)$
- The expected value is $E(X) = n \cdot p$

Exercise

Send a 1024-bit word over the BSC with bit error probability 0.03*.*
What is the expected number of bit errors in the received word?