Channels with Memory

Statistical Dependence

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Independent and Dependent Events

- Statistical independent events A and B
  - $P(A) = P(A|B) = P(A|\neg B)$
  - $P(B) = P(B|A) = P(B|\neg A)$
- Product rule — only for independent events
  - $P(A) \cdot P(B) = P(A \wedge B)$

Bernoulli trials

- Bernoulli trials are independent
- A BSC(p) gives $\mathbf{r} = \mathbf{x} + \mathbf{e}$
    - $\mathbf{e} = (e_1, e_2, \ldots, e_n)$
- $P(e_i) = P(e_i | e_j)$ for any $j \neq i$

Observing an error in position i does not change the probability distribution on position j.

A channel with memory

On a channel with memory the error probability at time t, depends on the occurrence of previous errors.

- Consider a distribution for $\mathbf{e} = (e_1, e_2, \dots, e_n)$
- $P(e_1 = 1) = 0.1$, $P(e_1 = 0) = 0.9$
- For $i > 1$,
  - $P(e_i = 0|e_{i-1} = 0) = 0.1$, $P(e_i = 1|e_{i-1} = 0) = 0.9$
  - $P(e_i = 0|e_{i-1} = 1) = 0.2$, $P(e_i = 1|e_{i-1} = 1) = 0.8$

One bit memory. The channel remembers what happened one bit past.

Summary

- Statistical independence is crucial
  - independent events simplify many formulæ
  - enables rigourous analysis
- Dependent events give more alternatives
  - complex (often intractible) analysis
- Independent events are prerequisites for
  - the central limit theorem
  - the binomial distribution

Exercise

Implement a simulator generating random 10-bit error vectors with the probability distribution of the previous slide. Run a simulation and estimate the mean number of bit errors per 10-bit word. Run enough tests to get a 95% *confidence interval smaller than* ± 0.3.