The Distribution of the Error Rate

The Normal Distribution

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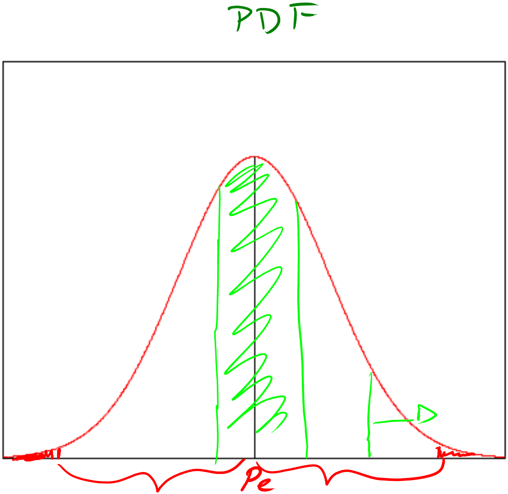
- **Objective:** Estimate the error probability p_e
- **Method:** Test the system n times
  - Record the number of errors X
- **Output:** Point estimator $\boxed{\hat{p}_e = X/n}$

Probability Distribution

$\hat{p}_e$



Central Limit Theorem

- The Binomial variable $X \sim B(n, p)$
  - is a sum $X = X_1 + X_2 + \ldots + X_n$
  - each $X_i \in \{0, 1\}$ is a Bernoulli trial with success probability p
- Central Limit Theorem
  - any sum $X = X_1 + X_2 + \ldots + X_n$
  - of identically distributed variables X_i
  - regardless of the exact distribution of X_i
  - as $n \to \infty$, X has always the same distribution
- This distribution, at the limit at infinity, is known as
  - the normal distribution, or
  - the Gaussian distribution

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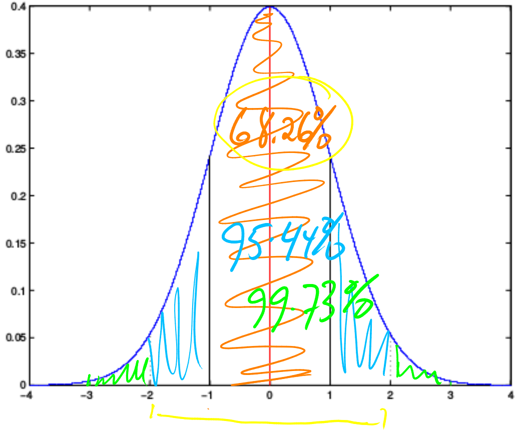
The Gauss Curve

The PDF of the standard normal distribution



PDF

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu = 0$

$\sigma = 1$

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Summary

- Let $X = X_1 + X_2 + \ldots + X_n$
  - sum of n identically distributed variables X_i
- When $n \to \infty$, $X \sim \boxed{N(\mu, \sigma)}$ — X is normally distributed

Exercise

Find the following probabilities using either software (e.g. Matlab) or a z-table (e.g. in Frisvold and Moe):

1. $P(0 \leq Z \leq 1)$ *when* $Z \sim N(0, 1)$
2. $P(-0.5 \leq Z \leq +0.5)$ *when* $Z \sim N(0, 1)$
3. $P(2 \leq Z \leq 5)$ *when* $Z \sim N(3, 2)$

Note that a z-table only gives $N(0, 1)$. For other values of μ and σ, you need to transform the variable and consider $\boxed{Z' = (Z - \mu)/\sigma.}$ See the textbook.