The Variance

The Binomial Distribution

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The Binomial Distribution

$$P(T = t) = \binom{n}{t} p^t (1 - p)^{n-t}$$

Problem

What is the variance $\mathrm{var}(T)$ *where the probability distribution of* T *is given above.*

Toy case $N = 1$

- Consider a single Bernoulli trial.
  - $X \sim B(1, p)$
  - $\mu = E(X) = p$
- What is the variance $\mathrm{var}(X)$
  - $\mathrm{var}(X) = \sum_x (x - \mu)^2 \cdot P(X = x)$

Outcome	X	$(\mu - X)^2$	Probability p'	$p' \cdot (\mu - X)^2$
Success	1	$(p-1)^2$	p	$p(p-1)^2$
Failure	0	p^2	$1-p$	$(1-p)p^2$
			Sum	$p(1-p)$

General case $N = ?$

- Binomial distribution $Y \sim B(n, p)$
- $Y = X_1 + X_2 + \ldots + X_n$
  - Each $X_i \sim B(1, p)$
  - Independent X_i
- $\mathrm{var}(Y) = \sum_{i=1}^{n} \mathrm{var}(X_i) = n \cdot \mathrm{var}(X) = n \cdot p(1 - p)$

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Summary

- Binomial distribution $X \sim B(n, p)$
- The expected value is $E(X) = n \cdot p$
- The expected value is $\mathrm{var}(X) = n \cdot p(1 - p)$

Exercise

Let T be the number of bit errors when an n-bit word is transmitted over BSC with bit error probability p. What is the variance $\mathrm{var}(T)$ when ...

1. $n = 7$
2. $n = 1024$

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