Words on the Channel

The error word

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The Word

- Long sequences of bits
- Split the sequence into blocks, or words
- Fixed-length words

Definition (Word)

An n-bit word is an n-dimensional vector, i.e. an element of $\mathbb{Z}_2^n$.

A word on the BSC

- Transmitted word $\mathbf{x}$
- Received word $\mathbf{r}$

$$\mathbf{r} = \mathbf{x} \oplus \mathbf{e} \tag{1}$$

- Error word $\mathbf{e} = (e_1, e_2, \ldots, e_n)$
  - $e_i = 1$ with probability p (error)
  - $e_i = 0$ with probability $1 - p$ (correct bit)

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Hamming weight

- How many bit errors do we have in a word?
- Let t be the number of errors
- $t = w(\mathbf{e})$ is the number of one-bits in the error word

Definition (Hamming weight)

The Hamming weight $w(\mathbf{x})$ of a vector $\mathbf{x}$ is the number of non-zero elements of $\mathbf{x}$.

Stochastic variables

- We had the channel formula
  - $\mathbf{r} = \mathbf{x} \oplus \mathbf{e}$
- Considering the error as a stochastic variable, we should write
  - $\mathbf{R} = \mathbf{x} \oplus \mathbf{E}$
- The number of errors is $T = w(\mathbf{E})$, $0 \leq T \leq n$
  - another stochastic variable
- What is the distribution of T?

Closure

Problem

Let T be the number of bit errors when transmitting an n-bit word over a BSC with bit error probabilty p.
Describe the probability distribution of T.

- The answer to this problem is known as the binomial distribution.
- We will introduce it in the next video.

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