Error Margin

Estimating the Mean with Known Variance

Prof Hans Georg Schaathun

Høgskolen i Ålesund

11th February 2014

Estimating the Mean

- What is the mean μ?
  - Stochastic variable X
  - Known standard deviation σ
  - Unknown distribution
- The Sample Mean is a point estimator
  - n observations: $x_1, x_2, \ldots, x_n$
  - Sample mean: $\hat{\mu} = \bar{x} = \sum x_i/n$
- How do we make a confidence interval?

HØGSKOLEN I ÅLESUND
Aalesund University College

Probability distribution

What is the probability distribution of $\bar{X}$?

$$\bar{X} = \overbrace{\frac{X_1}{n} + \frac{X_2}{n} + \ldots + \frac{X_n}{n}}$$

Variable	Expected value	Variance	Std.Dev.
X_i	μ	σ^2	σ
$\frac{X_i}{n}$	$\frac{\mu}{n}$	$\left(\frac{\sigma}{n}\right)^2$	$\frac{\sigma}{n}$
$\bar{X}$	$n \cdot \frac{\mu}{n}$ =	$n \cdot \left(\frac{\sigma}{n}\right)^2$ =	
$\bar{X}$	μ	$\frac{\sigma^2}{n}$	$\boxed{\frac{\sigma}{\sqrt{n}}}$

Central Limit Theorem: For large n, $\bar{X}$ has normal distribution.

HØGSKOLEN I ÅLESUND
Aalesund University College

Estimation Error

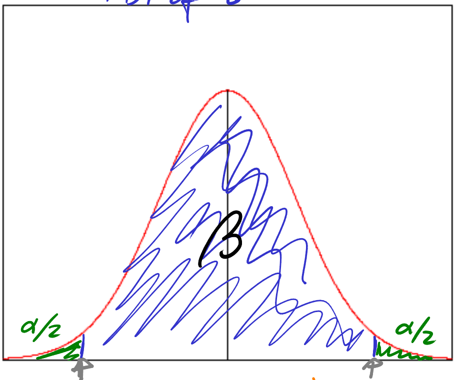
- Estimation error $\boxed{E = \bar{X} - \mu}$
  - $E \sim N(0, \sigma/\sqrt{n})$
- Normalised $\boxed{Z = \frac{E}{\sigma/\sqrt{n}}} = \boxed{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}}$
  - $\boxed{Z \sim N(0, 1)}$

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$

Error Distribution

$1-\beta=\alpha$

PDF of E



$\cancel{z=-1.6}$

$z=-1.96$

$e = 1.96 \cdot \sigma/\sqrt{n}$

- $\boxed{E = Z \cdot \sigma/\sqrt{n}}$
- $P(Z \leq -z) = \alpha/2$
- $P(E \leq -e) = \alpha/2$

$\alpha/2$	z	e
2.5%	1.96	$1.96 \cdot \sigma/\sqrt{n}$
1.0%	2.33	$2.33 \cdot \sigma/\sqrt{n}$
0.5%	$\cancel{2.33}$ 2.58	$2.58 \cdot \sigma/\sqrt{n}$

- We write $z_{\alpha/2}$

Matlab: `z = -icdf('norm', α/2, 0, 1)`

Summary

- Error $\boxed{E = \bar{X} - \mu}$
- $z_{\alpha/2}$ is such that $P(Z \leq -z) = P(Z \geq z) = \alpha/2$
- $P(E \leq -z_{\alpha/2} \cdot \sigma/\sqrt{n}) = P(E \geq z_{\alpha/2} \cdot \sigma/\sqrt{n}) = \alpha/2$
  - `z = -icdf('norm', α/2, 0, 1)`
- $P(|E| \geq z_{\alpha/2} \cdot \sigma/\sqrt{n}) = \alpha$
- With probability $\beta = 1 - \alpha$ (confidence level)
  - the error is within $\pm z_{\alpha/2} \cdot \sigma/\sqrt{n}$