The Sample Mean

Point Estimation by Example

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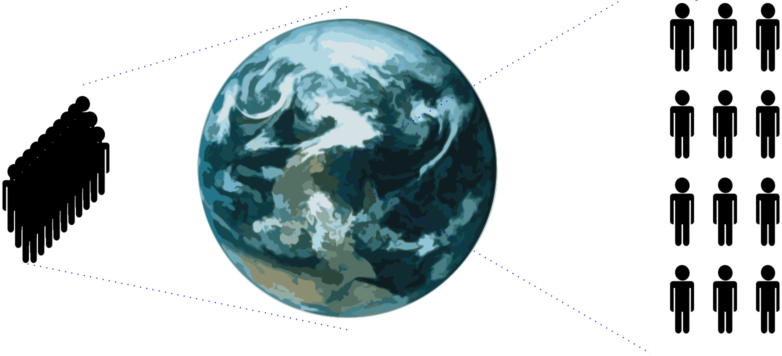
Sample and Population Mean

Population Mean

Sample Mean



$\mu = \frac{1}{\#E} \sum_{i \in E}^{n} x_i$

$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

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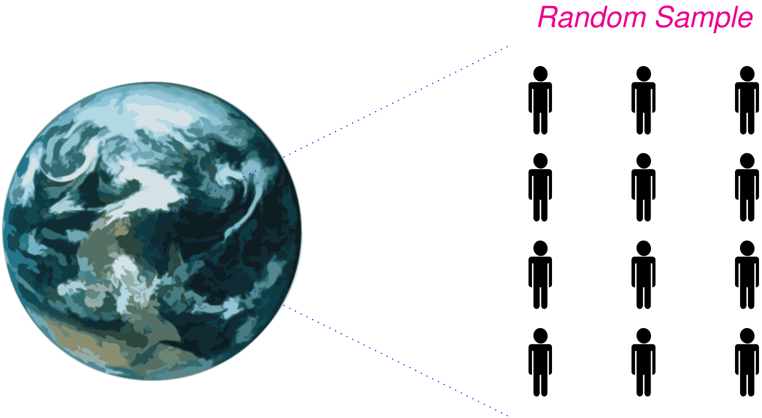

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

A statistic

Definition (Bhattacharyya and Johnson)

A statistic is a function of the sample observations.

- $\bar{x} = f(x_1, x_2, \ldots, x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$
- Each observation is the result of a random (stochastic) variable
  - $X_i \to x_i$
- A function of a stochastic variables is a new stocastic variable
  - $\bar{X} = f(X_1, X_2, \ldots, X_n) = \frac{1}{n}\sum_{i=1}^{n} X_i$

Remember the sample mean is a stochastic variable.

The mean of the mean

Question

What is the expected value (mean) $E(\bar{X})$?

1. Remember definition: $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
2. X_i are independent $\Rightarrow E(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n} E(X_i)$
3. Identical distribution $\Rightarrow E(\bar{X}) = \frac{1}{n} \cdot n \cdot \mu = \mu$

The expected value of the sample mean is equal to the population mean.

The mean of the mean

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Estimation

To estimate to guess, roughly judge or calculate

Approximation $x \approx y$ means $|x - y|$ is small

Estimation x is an estimate of y if $|x - y|$ probably is small

Definition (Estimator)

An estimator $\hat{\theta}$ of a parameter θ is a statistic (function of sample observations) used to estimate θ.

- Applying the function $\hat{\theta}$ gives an estimate of θ

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Sample mean as an estimator

Definition (Unbiased estimator)

An estimator $\hat{\theta}$ is said to be unbiased if $E(\hat{\theta}) = \theta$.

Proposition

The sample mean $\bar{X}$ is an unbiased estimator of the population mean μ.

- $\bar{X}$ is a stochastic variable
- The observed value $\bar{x}$ is an estimate of μ