The Standard Error

The Random Nature of Estimators

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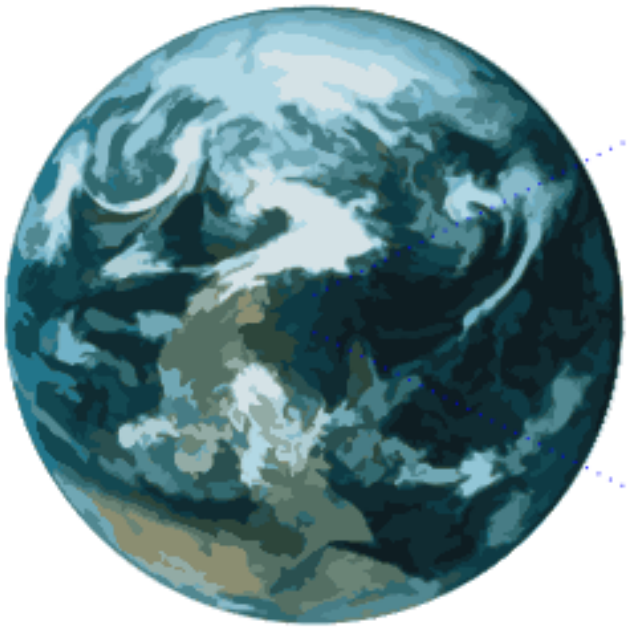
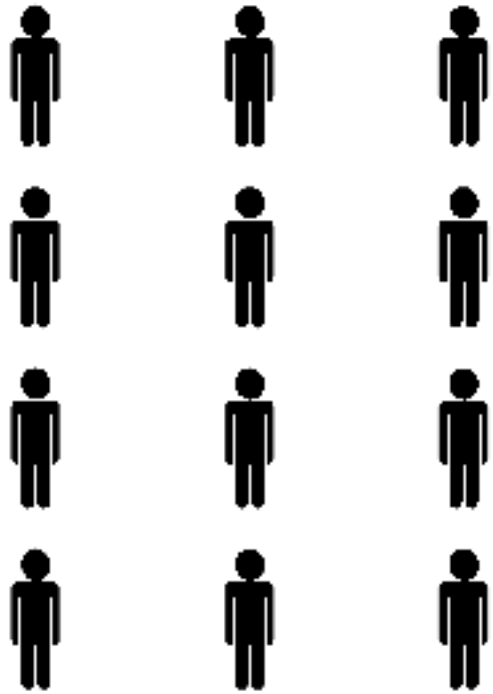
Sample Mean



Random Sample



$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

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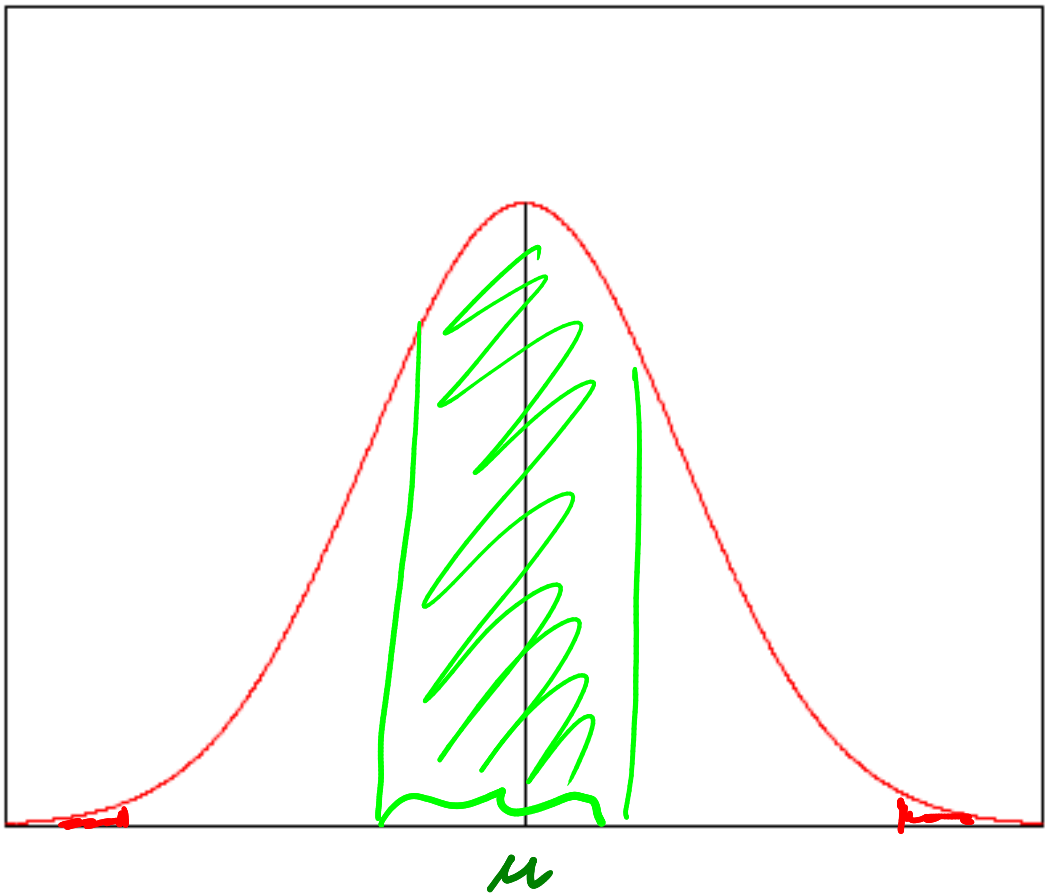
Probability Distribution

PDF for $\bar{x}$



The Standard Error

- Estimator $\hat{\theta}$
  - Stochastic variable
  - Probability distribution
  - Mean $E(\hat{\theta})$
  - Variance $\mathrm{var}(\hat{\theta})$

Definition

The standard deviation σ of an estimator $\hat{\theta}$ is called the standard error.

- We write $\mathrm{S.E.}(\hat{\theta})$

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The Standard Error of the Sample Mean

Step 1: Variance

Question

What is the standard error $\text{S.E.}(\bar{X})$*?*

$$\begin{aligned}
\text{var}(\bar{X}) &= \text{var}\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) \\
&= \frac{1}{n^2}\text{var}\left(\sum_{i=1}^{n} X_i\right) \\
&= \frac{1}{n^2}\sum_{i=1}^{n} \text{var}(X_i) \\
&= \frac{1}{n^2} \cdot n \cdot \sigma^2 = \boxed{\sigma^2/n.}
\end{aligned}$$

The Standard Error of the Sample Mean

Step 2: The Standard Error

Question

What is the standard error $\text{S.E.}(\bar{X})$*?*

$$\begin{aligned}\text{S.E.}(\bar{X}) &= \sqrt{\text{var}(\bar{X})} \\ &= \sqrt{\sigma^2/n} \\ &= \frac{1}{\sqrt{n}}\sigma.\end{aligned}$$

Summary

1. Estimators are stochastic variables
2. The standard deviation of an estimator: Standard Error
3. $\text{S.E.}(\bar{X}) = \sigma/\sqrt{n}$
   - where σ is std. deviation of X
4. Larger samples (n) gives smaller standard error

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