Estimating the Mean from Small Samples

Student's t-Distribution

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Confidence intervals with small samples

$$\bar{X} - z_{\alpha/2} \cdot \sigma_{\bar{X}} \leq \mu \leq \bar{X} + z_{\alpha/2} \cdot \sigma_{\bar{X}}$$

- Assumes large samples
- We introduce Student's t-distribution

Normalisation

$$X \sim N(\mu, \sigma)$$

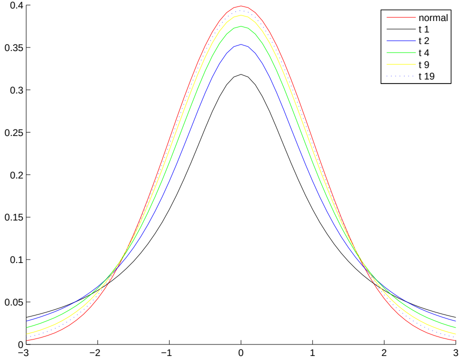
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \quad \text{where} \quad Z \sim N(0, 1)$$

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \quad \text{where} \quad T \sim T(n - 1)$$

- $T(\nu)$ — t-distributions with $\nu = n - 1$ degrees of freedom

When ν (n) is very large, the t-distribution is identical to the standard normal distribution.

The Probability Distribution



A new confidence interval

Summary

1. Estimate μ with known σ

$$\bar{X} - z_{\alpha_2} \cdot \sigma/\sqrt{n} \leq \mu \leq \bar{X} + z_{\alpha_2} \cdot \sigma/\sqrt{n}$$

2. Estimate μ with unknown σ

$$\bar{X} - t_{\alpha_2}^{(n-1)} \cdot s/\sqrt{n} \leq \mu \leq \bar{X} + t_{\alpha_2}^{(n-1)} \cdot s/\sqrt{n}$$

3. Matlab: `t = icdf( 't',` $\alpha/2$`, n-1 )`
4. Note, X must be normally distributed to use the t-distribution.
5. When n is large X may have any distribution.

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