The Alternative Hypothesis

One-sided or Two-sided Test?

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20th March 2014

The Null Hypothesis

- H_0 : The error probability p_e is 10^{-9}
  - $p = 10^{-9}$
- What is the alernative to H_0 being true?
- What are we really trying to test?
- We write H_1 for the alternative hypothesis
  1. $H_1 : p_e \neq 10^{-9}$
  2. $H_1' : p_e < 10^{-9}$

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Two-Sided Tests

A logical viewpoint

$$H_0 : p_e = 10^{-9}$$
$$H_1 : p_e \neq 10^{-9}$$

- The two hypotheses are logical negations
- This is similar to previous videos
  - we reject H_0 if
    1. the test statistic is large
    2. the test statistic is small
- This is called a two-sided test

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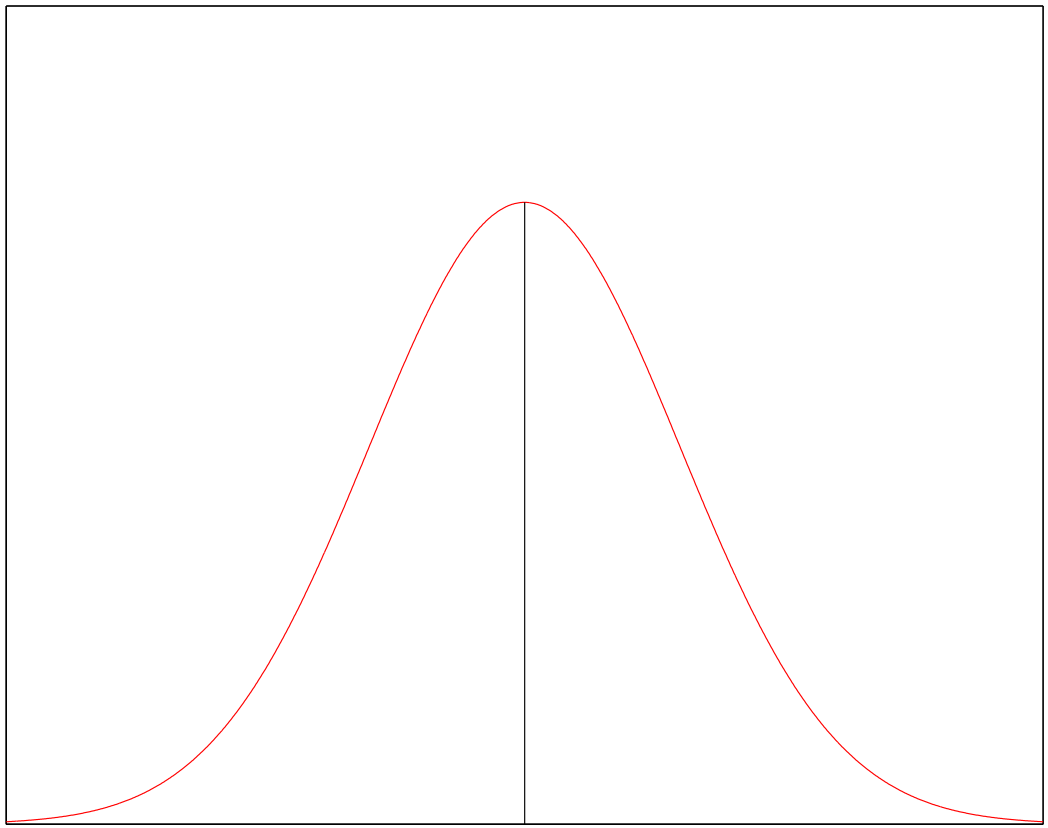
The probability distribution



The Practical Viewpoint

- We want to demonstrate $p_e < 10^{-9}$
- Two competing hypotheses
  - $A : p_e < 10^{-9}$
  - $B : p_e \geq 10^{-9}$
- What can be the null hypothesis H_0?
- $H_0 : p_e = 10^{-9}$
  - Equality gives known probability distribution
- We can choose the alternative hypothesis
  1. Two-sided test: $H_1 : p_e \neq 10^{-9}$
  2. One-sided test: $H_1 : p_e < 10^{-9}$

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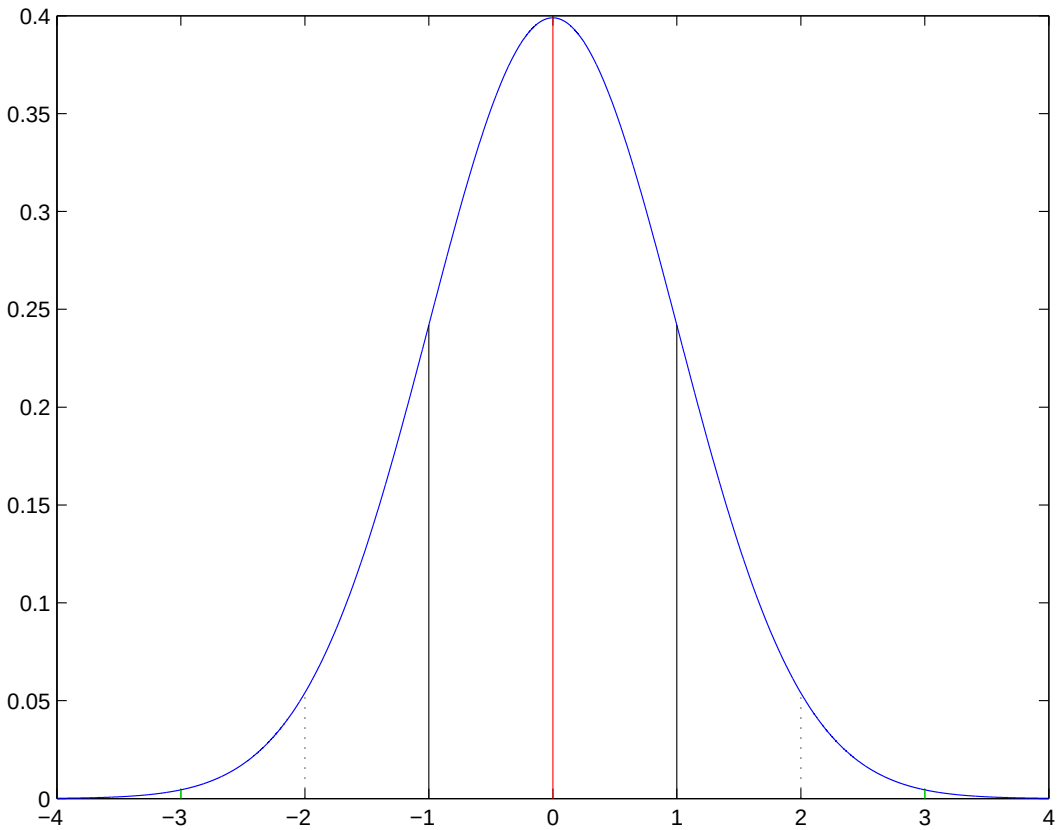

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Summary

- $H_0 : \mu = \mu_0$; σ known
- Test on population mean
  - use sample mean $\bar{X}$
- We normalise
  - $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$
  - $Z \sim N(0,1)$ under H_0
- Two-sided test — $H_1 : \mu \neq \mu_0$
  - Reject if $|Z| > z_{\alpha/2}$ where $P(Z > z_{\alpha/2}) = \alpha/2$
- One-sided test — $H_1 : \mu > \mu_0$
  - Reject if $Z > z_\alpha$ where $P(Z > z_\alpha) = \alpha$

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