Testing with the Binomial Distribution

Binomial Proportions with Small Sample

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Example Problem

H_0 : engineering students have 30% probability of failing on the elective maths module.

1. Check n students from the database
2. Count the number of failures X
3. $X \sim B(n, \pi)$
4. X can be used as a test statistic.

- Now, suppose n is relatively small.
- We cannot use the normal approximation

The Binomial Distribution

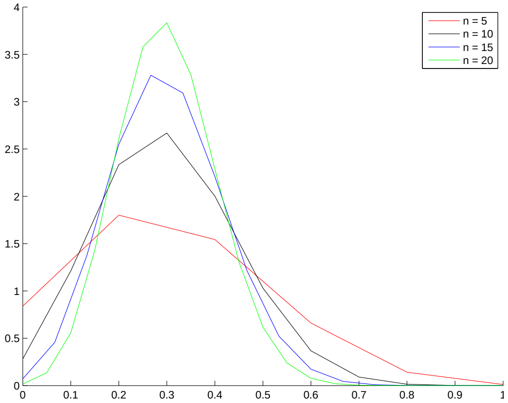
$$P(X = x) = \binom{n}{x} \cdot \pi^x \cdot (1 - \pi)^{n-x}, \tag{1}$$

$$X \sim B(n, \pi) \tag{2}$$

- Cumulative Distribution Function
    - $P(X \leq x)$
    - $1 - P(X > x)$
- Use probability tables for the binomial distribution,
    - or matlab or another software tool

The Probability Distribution Function

$X \sim B(n, 0.3)$



Calculating the p-value

The one-sided case

- $H_0 : \pi = \pi_0$ versus $H_1 : \pi > \pi_0$
    - $p = P(X \geq x_{\text{obs}})$
- $H_0 : \pi = \pi_0$ versus $H_1 : \pi < \pi_0$
    - $p = P(X \leq x_{\text{obs}})$

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Calculating the p-value

The two-sided case

- $H_0 : \pi = \pi_0$ versus $H_1 : \pi \neq \pi_0$
  - $p = 2 \cdot P(X \geq x_{\text{obs}})$ when $x_{\text{obs}} > n\pi$
  - $p = 2 \cdot P(X \leq x_{\text{obs}})$ when $x_{\text{obs}} < n\pi$

Summary

- Hypothesis test with a binomial distribution
  - $H_0 : \pi = \pi_0$
- If the sample is small, use binomial distribution
  - $P(X = x) = \binom{x}{n} \cdot p^x \cdot (1 - p)^{n-x}$
- $H_0 : \pi = \pi_0$ versus $H_1 : \pi > \pi_0$
  - $p = P(X \geq x_{\text{obs}})$
- $H_0 : \pi = \pi_0$ versus $H_1 : \pi < \pi_0$
  - $p = P(X \leq x_{\text{obs}})$
- $H_0 : \pi = \pi_0$ versus $H_1 : \pi \neq \pi_0$
  - $p = 2 \cdot P(X \geq x_{\text{obs}})$ when $x_{\text{obs}} > n\pi$
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