Hypothesis Testing and Confidence Intervals

A first interpretation of the test

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Testing a Hypothesis

- H_0: The error probability is 1%
- In other words
  - number of errors X over n tests
  - $X \sim B(n, p)$
  - $H_0 : p = 0.01$
- We want to test the hypothesis
  - Maximum 5% probability of Type I error

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The Level of Significance

- Remember, Type I errors are serious
- Bound the probability of such errors
  - $P(\text{Type I}) \leq \alpha$
- The bound α is called the level of significance

In the example $\alpha = 0.05$

Interval Estimation

$$\hat{p} - z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Confidence interval $(\hat{p}_0, \hat{p}_1)$
  - Level of confidence $\beta = 1 - \alpha = 95\%$
  - With 95% probability $\hat{p}_0 \leq p \leq \hat{p}_1$
- If H_0 is true
  1. $p = 0.01$
  2. With 95% probability $\hat{p}_0 \leq 0.01 \leq \hat{p}_1$
- Run the test
  1. If $\hat{p}_0 \leq 0.01 \leq \hat{p}_1$, then H_0 plausible
  2. If $0.01 < \hat{p}_0$ or $0.01 > \hat{p}_1$, then H_0 is not plausible
- Significance level $(100 - 95)\% = 5\%$

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Confidence and Significance

- Confidence level β for estimation
- Significance level α for hypothesis testing
- Connection $1 = \alpha + \beta$

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Summary

- Strong connection
  1. Hypothesis testing
  2. Confidence intervals
- Hypothesised value is expected to be inside the confidence interval
- If it is not, we reject the null hypothesis
- Confidence level β related to significance level α
  - $\alpha = 1 - \beta$

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