

Hypothesis Testing

Introduction

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Hypothesis Testing

| Product | Claim |
|---------------|---|
| Drug | Patients recover better with drug, than without it. |
| Coding system | Error rate is less than 10^{-9} . |
| New road | Travel times will be reduced by at least 10 min. |

The Fundamental Challenge

How do we test a hypothesis?

- 1 Define a null hypothesis.
- 2 Observation. Gather data.
- 3 Are the observed data plausible if the hypothesis be true?
 - Not plausible data. Reject the null hypothesis.
 - Plausible data. Keep the null hypothesis.

How do we assess plausibility?

The test statistic

Assessing plausibility

- 1 Test statistic S
 - observable quantity (e.g. sample mean)
 - known probability distribution under the null hypothesis
- 2 Known expectation $E(S)$ under the null hypothesis H_0
- 3 Observed $s \approx E(S) \Rightarrow$ plausible null hypothesis
- 4 Observed $s \not\approx E(S) \Rightarrow$ implausible null hypothesis

For example — the mean

Does the average student drink more than two pints of beer on a Saturday?

- 1 $H_0: \mu = 2\text{pints}$
- 2 Sample statistic: \bar{x}
- 3 If H_0 , then $E(\bar{x}) = 2\text{pints}$
- 4 If $\bar{x} \gg 2\text{pints} \Rightarrow H_0$ implausible
- 5 If $\bar{x} \ll 2\text{pints} \Rightarrow H_0$ implausible
- 6 If $\bar{x} \approx 2\text{pints} \Rightarrow H_0$ plausible

Summary

- 1 Define null hypothesis H_0
- 2 Define test statistic with known probability distribution under H_0
- 3 Gather data, get an observation s of S
- 4 Is the observed s plausible under H_0 ?
 - Not plausible. Reject the null hypothesis.
 - Plausible. Keep the null hypothesis.

In later videos, we will discuss how quantify the plausibility.