

The Mean with Known Variance

An example of hypothesis testing

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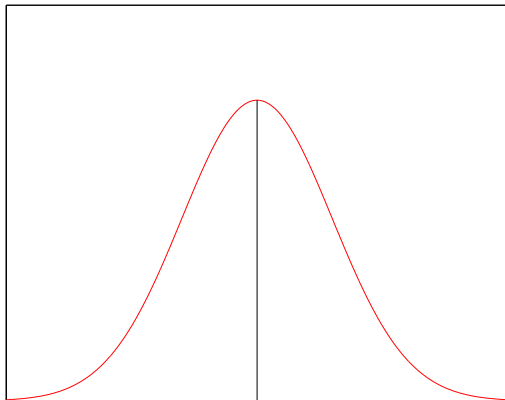
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Example Problem

Claim The average student drinks two pints of beer on Saturday.

- The claim is a hypothesis
 - $H_0 : \mu = 2$
- We can poll n students
 - How many pints of beer do you drink on Saturday?
 - Observations: $X_1, X_2, X_3, \dots, X_n$
- We assume $\sigma_X = 1.5$
- Test statistic:
 - Sample mean \bar{X}

The probability distribution



Simplification

We need to know the probability distribution of \bar{X} under H_0

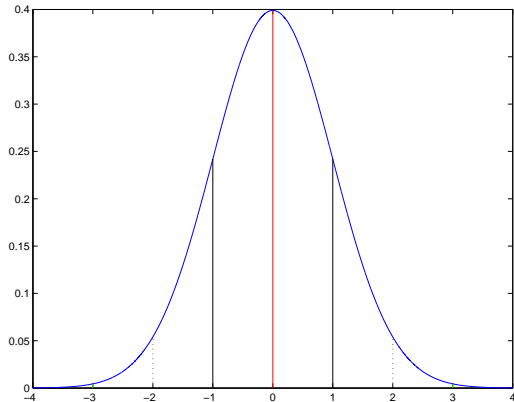
- Normal distribution when n is large
 - Central Limit Theorem
- $\bar{X} \sim N(\mu, \sigma)$
- $\mu = 2, \sigma_X = 1.5$
 - $\sigma_{\bar{X}} = \sigma_X / \sqrt{n} = 1.5 / \sqrt{n}$

We leave unknown σ for the next video.

Normalisation

- $\bar{X} \sim N\left(2, \frac{1.5}{\sqrt{n}}\right)$
- Normalisation
 - 1 Subtract μ to get $\mu' = 0$
 - 2 Divide by σ to get $\sigma' = 1$
- $Z = \frac{\bar{X}-2}{1.5/\sqrt{n}}$
 - $Z \sim N(0, 1)$
- Let's use Z as the **test statistic**

The probability distribution



Summary

- $H_0 : \mu = \mu_0 ; \sigma$ known
- Test on population mean
 - use sample mean \bar{X}
- We normalise
 - $Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$
 - $Z \sim N(0, 1)$ under H_0
- Reject H_0 with significance level α
 - if $|Z| > z_{\alpha/2}$ where $P(Z > z_{\alpha/2}) = \alpha/2$