Testing with Binomial Proportions

The Case with Large Sample

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Example Problem

- H_0 : The error probability p_e is 10^{-9}
  - $p_e = 10^{-9}$
- p_e is a binomial proportion

1. Run n tests of the system
2. Count the number of errors X
3. $X \sim B(n, p_e)$
4. X can be used as a test statistic.

Recall the Confidence Interval

$$\hat{p} - z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $\hat{p}_e = X/n$.

- When n is large $\hat{p}_e \sim N\left(p_e, \sqrt{\frac{p_e(1-p_e)}{n}}\right)$
  - using the Central Limit Theorem
- Estimate σ_X using $\hat{p}_e$ and the formula for σ for a binomial distribution
- Otherwise similar to means for a normal distrubution

Normalisation

- $X \sim N(n \cdot p'_e, \sqrt{n \cdot p'_e \cdot (1 - p'_e)})$ under H_0
  - $p'_e = 10^{-9}$
- Normalisation
  1. Subtract the mean
  2. Divide by the standard deviation
- $Z = \frac{X - n \cdot p'_e}{\sqrt{n \cdot p'_e \cdot (1 - p'_e))}}$
  - $Z \sim N(0, 1)$
- We use Z as the test statistic

Summary

- Hypothesis test with a binomial distribution
  - H_0 : error probability is p'_e
- If the sample is large, use standard normal distribution:
  - $Z = \frac{X - n \cdot p'_e}{\sqrt{n \cdot p'_e \cdot (1 - p'_e)}}$
- Two-sided test — $H_1 : p_e \neq p'_e$
  - Reject if $|Z| > z_{\alpha/2}$ where $P(Z > z_{\alpha/2}) = \alpha/2$
- One-sided test — $H_1 : p_e > p'_e$
  - Reject if $Z > z_\alpha$ where $P(Z > z_\alpha) = \alpha$