Hypothesis Testing of the Mean

The case with unknown variance

Prof Hans Georg Schaathun

Høgskolen i Ålesund

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Example Problem

Claim *The average student drinks two pints of beer on Saturday.*

- The claim is a hypothesis
  - $H_0 : \mu = 2$
  - We know nothing about σ
- Polling students gives observations
  - $X_1, X_2, X_3, \ldots, X_n$
- Basis for a test statistic:
  - Sample mean $\bar{X}$

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Probability Distribution

We need to know the probability distribution of $\bar{X}$ under H_0

- Small n – cannot use the Central Limit Theorem
- Unknown σ
- What did we do with the confidence interval?

Student's t-distribution

- We need to assume that the X_i have Normal Distribution

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Normalisation and the t-distribution

$$T = \frac{\bar{X} - 2}{s/\sqrt{n}}$$

- s is the sample standard deviation
- $T \sim \mathcal{T}(n-1)$
- We can use T as the test statistic

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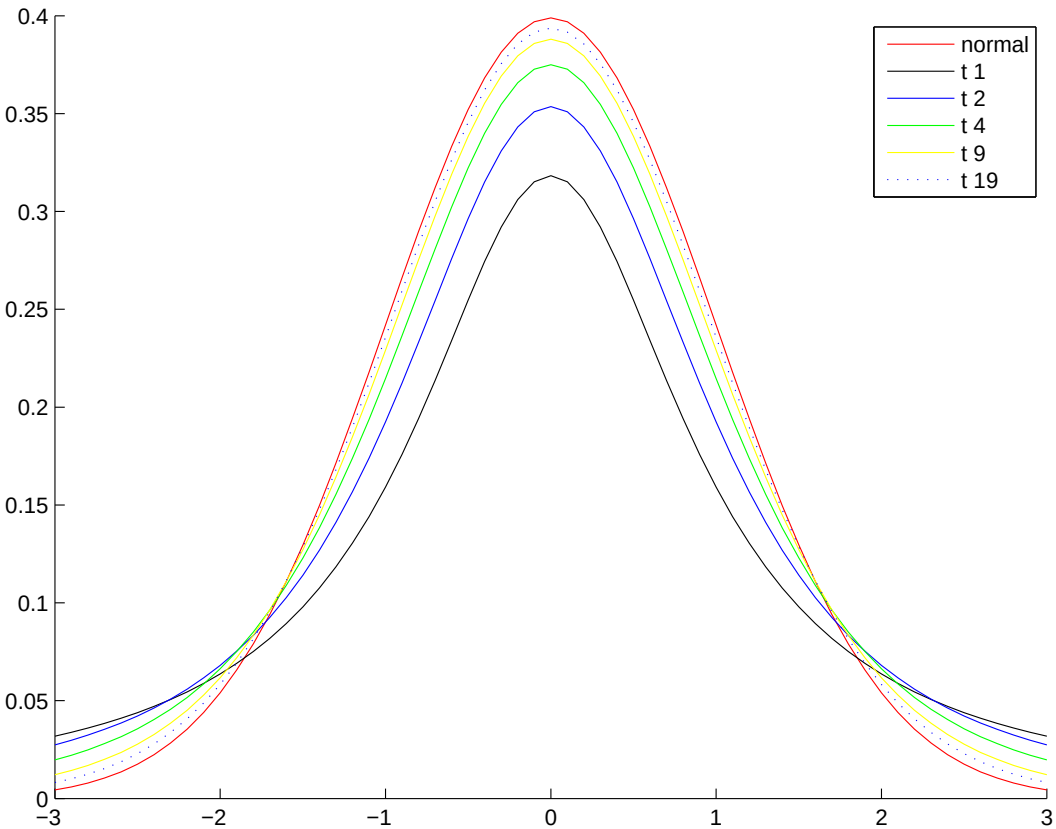
The probability distribution



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Summary

- $H_0 : \mu = \mu_0$; σ unknown
- Test on population mean
  - use sample mean $\bar{X}$
- We define a t-distributed test statistic
  - $T = \frac{\bar{X} - 2}{s/\sqrt{n}}$
  - $T \sim \mathcal{T}(n-1)$
- Reject H_0 with significance level α
  - (one-sided) if $T > t_{\alpha}^{n-1}$ where $P(T > t_{\alpha}^{n-1}) = \alpha$
  - (two-sided) if $|T| > t_{\alpha/2}^{n-1}$ where $P(T > t_{\alpha/2}^{n-1}) = \alpha/2$

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